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Orthogonal Jordan Triple Higher Reverse Left (resp. Right) Centralizer on Semiprime Rings

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Abstract

The aim of this article to offer the entry of orthogonal Jordan triple higher reverse left (resp. right) centralizers a ring ϒ & survey this concept.

Keywords: Semiprime ring (for short s.r); 2-torsion free;Orthogonal Jordan triple higher reverse left(resp. right) centralizers

1. Introduction

The connotation of orthogonal Jordan triple higher reverse left (resp.right)centralizers (for short O.J.T.H.R.L(resp.R))C is a substantial in non-commutative algebra.In[1] and[2] can see the definition of s.r and 2-torsion free (for short 2- tf) resp. of ring , approaching for more initiation [2&3] .

In this work ,define and study the concept of O.J.T.H.R.L(resp.R)C of s.r, need the next Lemmas :

1.1. Lemma (1.1): [2]

Let ϒ be a 2- tf s.r & ɤ, ϖ be elements of ϒ, then the next requirement are comparable

- ɤeϖ = 0, for each e ∈ ϒ
- ϖeɤ = 0, for each e ∈ ϒ
- ɤeϖ + ϖeɤ = 0, for each e ∈ ϒ

If one of these requirement is achieved, then ɤϖ = ϖɤ = 0

1.2. Lemma (1.2): [1]

Let ϒ be a 2- tf s.r & ɤ,ϖ be elements of ϒ if ɤeϖ + ϖeɤ = 0, each e ∈ ϒ,then ɤeϖ = ϖeɤ = 0

2. Orthogonal Jordan triple higher reverse left (resp.right) centralizer on Semiprime Rings

2.1. Definition (2.1):

Two J.T.H.R.L(resp.R)C $Ϫ = (Ϫ_i)_{i\in N}$ & $ϙ = (ϙ_i)_{i\in N}$ of a ringϒ are called orthogonal if$Ϫ_n$(ɤ)ϒ$ϙ_n$(ϖ) = (0) = $ϙ_n$(ϖ)ϒ$Ϫ_n$(ɤ), each ɤ , ϖ ∈ ϒ & n ∈ N .

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2.2. Lemma (2.2)

Let Υ be a 2- tf s.r $Ꭓ=(Ꭓ_i)_{i\in N}$ & $Ϙ=(Ϙ_i)_{i\in N}$ are two J.T.H.R.L(resp.R)C of Υ .Then $Ꭓ_n$ & $Ϙ_n$ are orthogonal if and only if $Ꭓ_n(ɤ)\, Ϙ_n(\varpi) + Ϙ_n(ɤ)\, Ꭓ_n(\varpi) = 0$, each $ɤ , \varpi \in \Upsilon$ & $n \in N$

Proof :

Imply $Ꭓ_n$ & $Ϙ_n$ are orthogonal

T.P. $Ꭓ_n(ɤ)\, Ϙ_n(\varpi) + Ϙ_n(ɤ)\, Ꭓ_n(\varpi) = 0$

Seeing that $Ꭓ_n$ & $Ϙ_n$ are orthogonal

$$\sum_{i=1}^{n} Ꭓ_i(ɤ)\, e\, Ϙ_i(\varpi) = 0 = Ϙ_i(\varpi)\, z\, Ꭓ_i(ɤ) \text{ , for each } ɤ , \varpi , e \in \Upsilon$$

As a consequence, through Lemma (1.1) , outcome

Conversely , imply $Ꭓ_n(ɤ)\, Ϙ_n(\varpi) + Ϙ_n(ɤ)\, Ꭓ_n(\varpi) = 0$, each $ɤ , \varpi \in \Upsilon$ & $n \in N$

T.P. $Ꭓ_n$ & $Ϙ_n$ are orthogonal

$$\sum_{i=1}^{n} Ꭓ_i(ɤ)\, Ϙ_i(\varpi) + Ϙ_i(ɤ)\, Ꭓ_i(\varpi) = 0$$

Left multiply by $Ꭓ_{i-1}(e)$

$$\sum_{i=1}^{n} Ꭓ_{i-1}(e)\, Ꭓ_i(ɤ)\, Ϙ_i(\varpi) + Ꭓ_{i-1}(e)\, Ϙ_i(ɤ)\, Ꭓ_i(\varpi) = 0$$

seeing that $Ꭓ_n$ & $Ϙ_n$ are commuting

$$\sum_{i=1}^{n} Ꭓ_i(ɤ)\, Ꭓ_{i-1}(e)\, Ϙ_i(\varpi) + Ϙ_i(ɤ)\, Ꭓ_{i-1}(e)\, Ꭓ_i(\varpi) = 0 \text{, for each } ɤ , \varpi , e \in \Upsilon$$

Through Lemma (1.2) , outcome

2.3. Theorem (2.3):

Let Υ be a 2- tf s.r ,$Ꭓ=(Ꭓ_i)_{i\in N}$ & $Ϙ=(Ϙ_i)_{i\in N}$ are two J.T.H.R.L(resp.R)C of Υ, where $Ꭓ_n$& $Ϙ_n$ are commuting. Then the next requirement are comparable, each $n \in N$:

- $Ꭓ_n$ & $Ϙ_n$ are orthogonal
- $Ꭓ_nϘ_n = 0$
- $Ϙ_n Ꭓ_n = 0$
- $Ꭓ_nϘ_n + Ϙ_n Ꭓ_n = 0$

Proof : (a) ⇔ (b)

Imply $Ꭓ_n$ & $Ϙ_n$ are orthogonal

T.P. $Ꭓ_nϘ_n = 0$, each $n \in N$

Seeing that $Ꭓ_n$ & $Ϙ_n$ are orthogonal

$$\sum_{i=1}^{n} Ϫ_i(ɤ)\ e\ Ϙ_i(ϖ) = 0 = \sum_{i=1}^{n} Ϙ_i(ϖ)\ e\ Ϫ_i(ɤ)\ \text{, each } ɤ , ϖ , e \in ϒ$$

$$(ɤ)) = 0_iϪ(_{1-i}Ϫ(e)\ _{1-i}Ϫ\ (ϖ))_i(Ϙ_iϪ \sum_{i=1}^{n}$$

As a substitute for $Ϫ_{i-1}(Ϫ_i(ɤ))$ by $Ϫ_i(Ϙ_i(ϖ))$

$$\sum_{i=1}^{n} Ϫ_i(Ϙ_i(ϖ))\ Ϫ_{i-1}(e)\ Ϫ_i(Ϙ_i(ϖ)) = 0$$

Seeing that ϒ is a s.r

$$\sum_{i=1}^{n} Ϫ_i(Ϙ_i(ϖ)) = 0, \text{ for each } ϖ \in ϒ \Rightarrow Ϫ_nϘ_n = 0, \text{ each } n \in N$$

Conversely, imply $ϪnϘn = 0$, each $n \in N$

T.P. $Ϫ_n$ & $Ϙ_n$ are orthogonal

$Ϫ_n(Ϙ_n(ϖeɤ)) = 0$

$$\sum_{i=1}^{n} Ϫ_i(Ϙ_i(ɤ))\ Ϫ_{i-1}(Ϙ_{i-1}(e))\ Ϫ_{i-1}(Ϙ_i(ϖ)) = 0 \text{ , each } ɤ , ϖ , e \in ϒ$$

As a substitute $Ϙ_i(ϖ)$ by $ɤ$

$$\sum_{i=1}^{n} Ϫ_i(ɤ)\ Ϫ_{i-1}(Ϙ_{i-1}(e))\ Ϫ_{i-1}(Ϙ_i(ϖ)) = 0$$

As a substitute $Ϫ_{i-1}(Ϙ_{i-1}(ϖ))$ by $Ϙ_i(ϖ)$

$$\sum_{i=1}^{n} Ϫ_i(ɤ)\ Ϫ_{i-1}(Ϙ_{i-1}(e))\ Ϙ_i(ϖ) = 0, \text{ each } ɤ , ϖ \in ϒ \ \dots (1)$$

Seeing that $Ϫ_n$ & $Ϙ_n$ are commuting

$$\sum_{i=1}^{n} Ϙ_i(ϖ)\ Ϫ_{i-1}(Ϙ_{i-1}(e))\ Ϫ_i(ɤ) = 0, \text{ each } ɤ , ϖ \in ϒ \ \dots (2)$$

Through (1) and (2) , outcome

Proof : (a) ⇔ (c)

Selfsame in (i) ⇔ (ii) , secure get (a) ⇔ (c) .

Proof : (a) ⇔ (d)

Imply $Ϫ_n$ & $Ϙ_n$ are orthogonal

By (ii) & (iii) , outcome

Conversely , imply $ѪnϘn + Ϙ_nѪ_n = 0$, each $n \in N$

T.P. $Ѫ_n$ & $Ϙ_n$ are orthogonal

$$\sum_{i=1}^{n} Ѫ_i(Ϙ_i(\omega e ɤ)) + Ϙ_i(Ѫ_i(\omega e ɤ)) = 0$$

$$\sum_{i=1}^{n} Ѫ_i(Ϙ_i(ɤ))\, Ѫ_{i-1}(Ϙ_{i-1}(e))\, Ѫ_{i-1}(Ϙ_{i-1}(\omega)) + Ϙ_i(Ѫ_i(ɤ))\, Ϙ_{i-1}(Ѫ_{i-1}(e))\, Ϙ_{i-1}(Ѫ_{i-1}(\omega)) = 0$$

As a substitute $Ϙ_i(ɤ)$ by $ɤ$ & $Ѫ_i(ɤ)$ by $ɤ$

$$\sum_{i=1}^{n} Ѫ_i(ɤ)\, Ѫ_{i-1}(Ϙ_{i-1}(e))\, Ѫ_{i-1}(Ϙ_i(ɤ)) + Ϙ_i(ɤ)\, Ϙ_{i-1}(Ѫ_{i-1}(e))\, Ϙ_{i-1}(Ѫ_i(ɤ)) = 0$$

As a substitute $Ѫ_{i-1}(Ϙ_i(ɤ))$ by $Ϙ_i(\omega)$ & $Ϙ_{i-1}(Ѫ_i(ɤ))$ by $Ѫ_i(\omega)$

$$\sum_{i=1}^{n} Ѫ_i(ɤ)\, Ѫ_{i-1}(Ϙ_{i-1}(e))\, Ϙ_i(\omega) + Ϙ_i(ɤ)\, Ϙ_{i-1}(Ѫ_{i-1}(e))\, Ѫ_i(\omega) = 0, \text{ each } ɤ, \omega, e \in Υ$$

Through Lemma (1.1) & Lemma (2.2), outcome

2.4. Theorem (2.4)

Let $Υ$ be a 2- tf s.r , $Ѫ =(Ѫ_i)_{i\in N}$ & $Ϙ = (Ϙ_i)_{i\in N}$ are two J.T.H.R.L(resp.R)C of $Υ$, imply $Ѫ^2{}_n = Ϙ^2{}_n$. Then $Ѫ_n + Ϙ_n$ & $Ѫ_n - Ϙ_n$ are orthogonal .

Proof :

$((Ѫ_n + Ϙ_n)(Ѫ_n - Ϙ_n) + (Ѫ_n - Ϙ_n)(Ѫ_n + Ϙ_n))(ɤ)$

$$= \sum_{i=1}^{n} ((Ѫ_i(ɤ) + Ϙ_i(ɤ))(Ѫ_i(ɤ) - Ϙ_i(ɤ)) + (Ѫ_i(ɤ) - Ϙ_i(ɤ))(Ѫ_i(ɤ) + Ϙ_i(ɤ)))$$

$$0 = \sum_{i=1}^{n} Ѫ^2{}_i(ɤ) - Ѫ_i(ɤ)Ϙ_i(ɤ) + Ϙ_i(ɤ)Ѫ_i(ɤ) - Ϙ^2{}_i(ɤ) + Ѫ^2{}_i(ɤ) + Ѫ_i(ɤ)Ϙ_i(ɤ) - Ϙ_i(ɤ)Ѫ_i(ɤ) - Ϙ^2{}_i(ɤ)$$

As a consequence, through Theorem(2.3)(iv) $\Rightarrow$ (i), outcome

3. Conclusion

In this paper we defined the concept of Orthogonal Jordan Triple Higher Reverse Left (resp. Right) Centralizer on Semiprime Rings we can find more of properties about this concept. According to the previous results, the future study in this paper will define Γ– rings and we can find more of Theorems and properties.

Compliance with ethical standards

Disclosure of conflict of interest

There was no conflict of interest

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